Abstract:
Mathematical Models that Measure our World

When we think of measurement, we usually think about measuring physical quantities such as temperature or length. But everyday, we come across measures for many other ‘quantities’ such as the star rating for healthiness of food, GDP, our BMI and measures of biodiversity, social inequality or the infectivity of a disease. All of these are mathematical models, used to understand and make predictions about the world. Defining a measure in mathematical terms (i.e., quantifying it) enables collection of data, identifying relationships to other quantities and importantly making predictions about the future.

Mathematical modelling is described in the Victorian Curriculum (Mathematics) 2.0 as an essential dimension of the contemporary discipline of mathematics and as a key to informed and participatory citizenship. But this requires a fuller appreciation of mathematical modelling than applying known techniques to already mathematised situations.

In this session, we will look at several examples of mathematical models that are commonly in everyday life or science, examine what makes the definition good, and discuss how students can come to understand how experts and citizens can use these mathematical models to make good decisions.

CURRICULUM, PEDAGOGY AND BEYOND

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Mathematical Models that Measure our World.

Kaye Stacey
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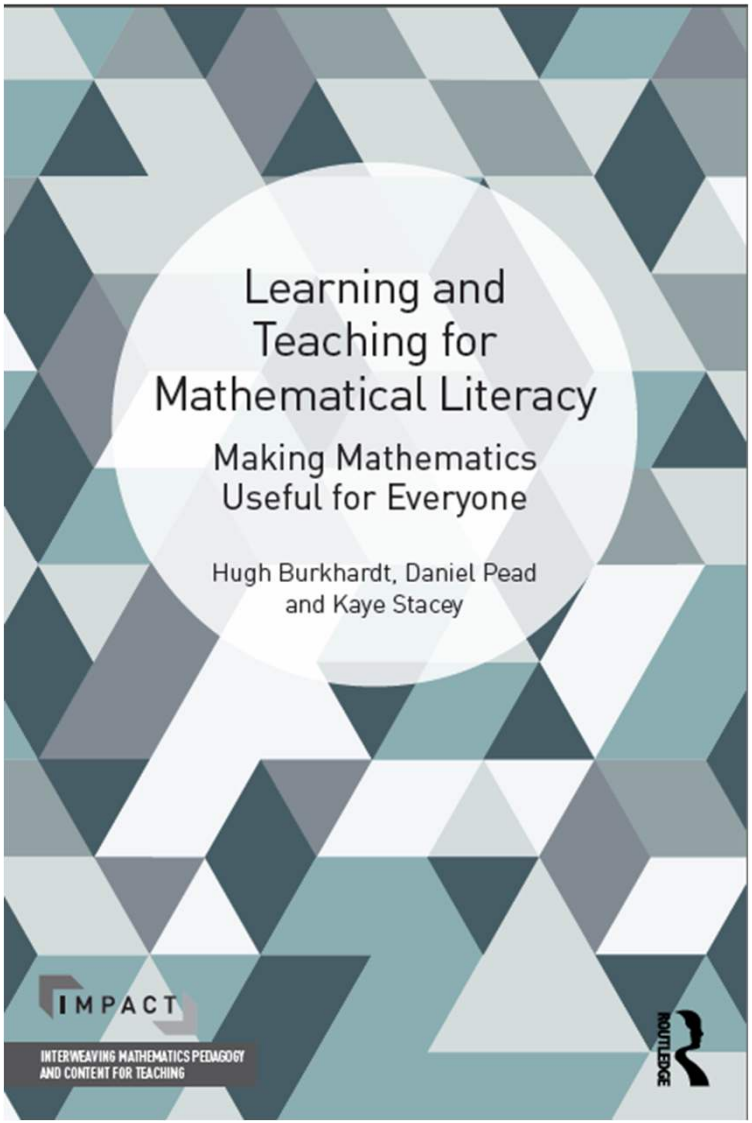


Making Mathematics Useful for Everyone

Mathematical literacy is a person's ability to use whatever mathematics they know in meaningful ways in a wide variety of out-of-school situations.

- Mathematical literacy is similar to the BIG version os numeracy, with real world problem solving and critical thinking
- Book describes the sort of teaching and learning that helps to justify the large amount of school time spent on mathematics.
- Many examples in text and in full through tdedicated website ltml.mathlit.org
- Focussed on ages 12 to adult

Four mathematical processes

Learning in Mathematics 2.0

- **Mathematical processes**
- Mathematical processes refer to the thinking, reasoning, communicating, problem-solving and investigation skills involved in working mathematically.
- Opportunities to learn process skills have been embedded across the strands, building in sophistication across the levels.
- Mathematical problem-solving and investigation draws on the processes of mathematical modelling, computational and algorithmic thinking, statistical investigation, probability experiments and simulations.
- **Mathematical modelling (one of four processes)**
- Mathematical models are used to gain insight into and make predictions about real-world phenomena, to inform judgements and make decisions in personal, civic and work life.
- In the modelling process students formulate a real-world problem mathematically by making assumptions; recognise, connect and apply mathematical structures; analyse and solve the mathematical model; and interpret, generalise and communicate their results in response to the real-world situation.
- Mathematical modelling is an essential dimension of the contemporary discipline of mathematics and is key to informed and participatory citizenship.

Modelling in the content descriptors

Year 8 VC2M8N06

use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts …; formulate problems, choosing efficient .. calculation strategies and using digital tools …; interpret and communicate solutions in terms of the context, reviewing the appropriateness of the model

Achievement standard. ….They use mathematical modelling to solve practical problems involving ratios, ….

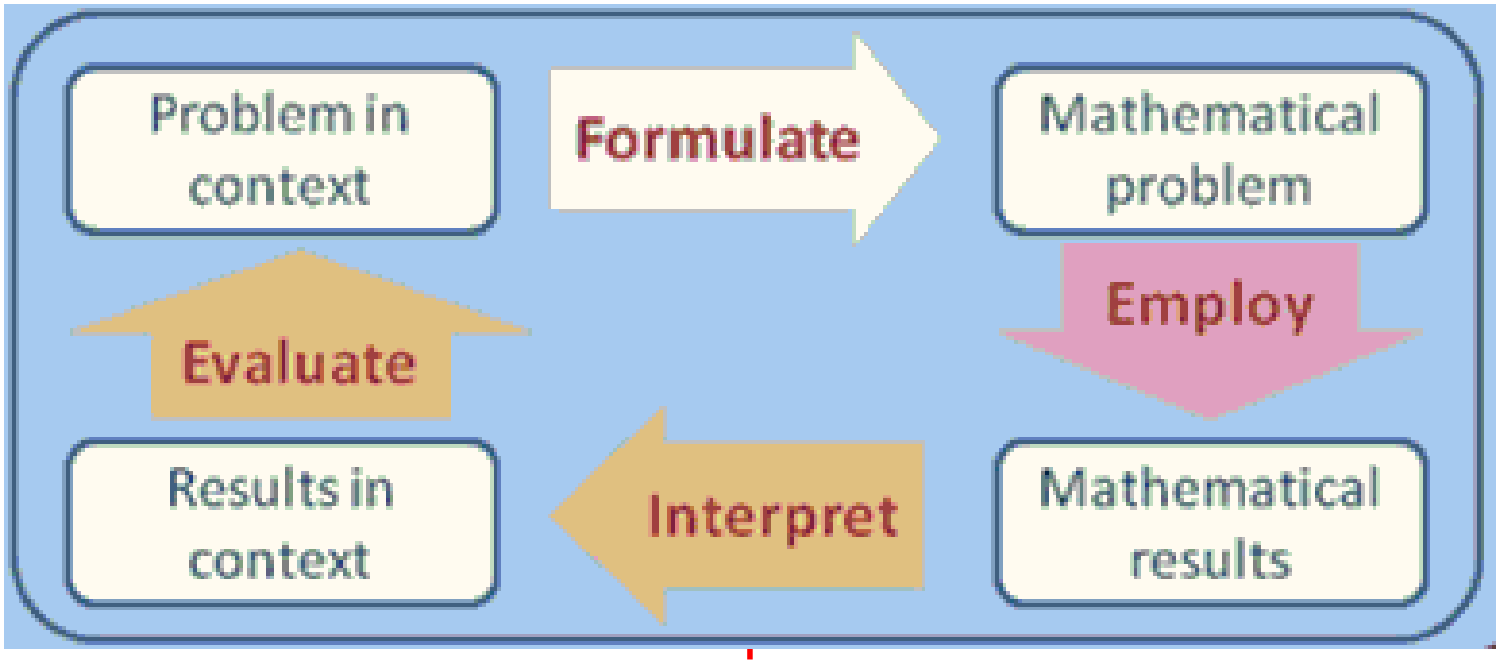
Year 10 VC2M10A15

use mathematical modelling to solve applied problems involving inverse proportion, growth and decay, including in financial contexts to establish the compound interest formula as repeated applications of simple interest; formulate problems, choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings

The modelling cycle

Thinking about real world ideas

Thinking about mathematical ideas



Topic chapters in LTML book

How risky is life?

Looking past the spin (adverts, politics etc)

Planning for enjoyment in life

Mathematics for information technology

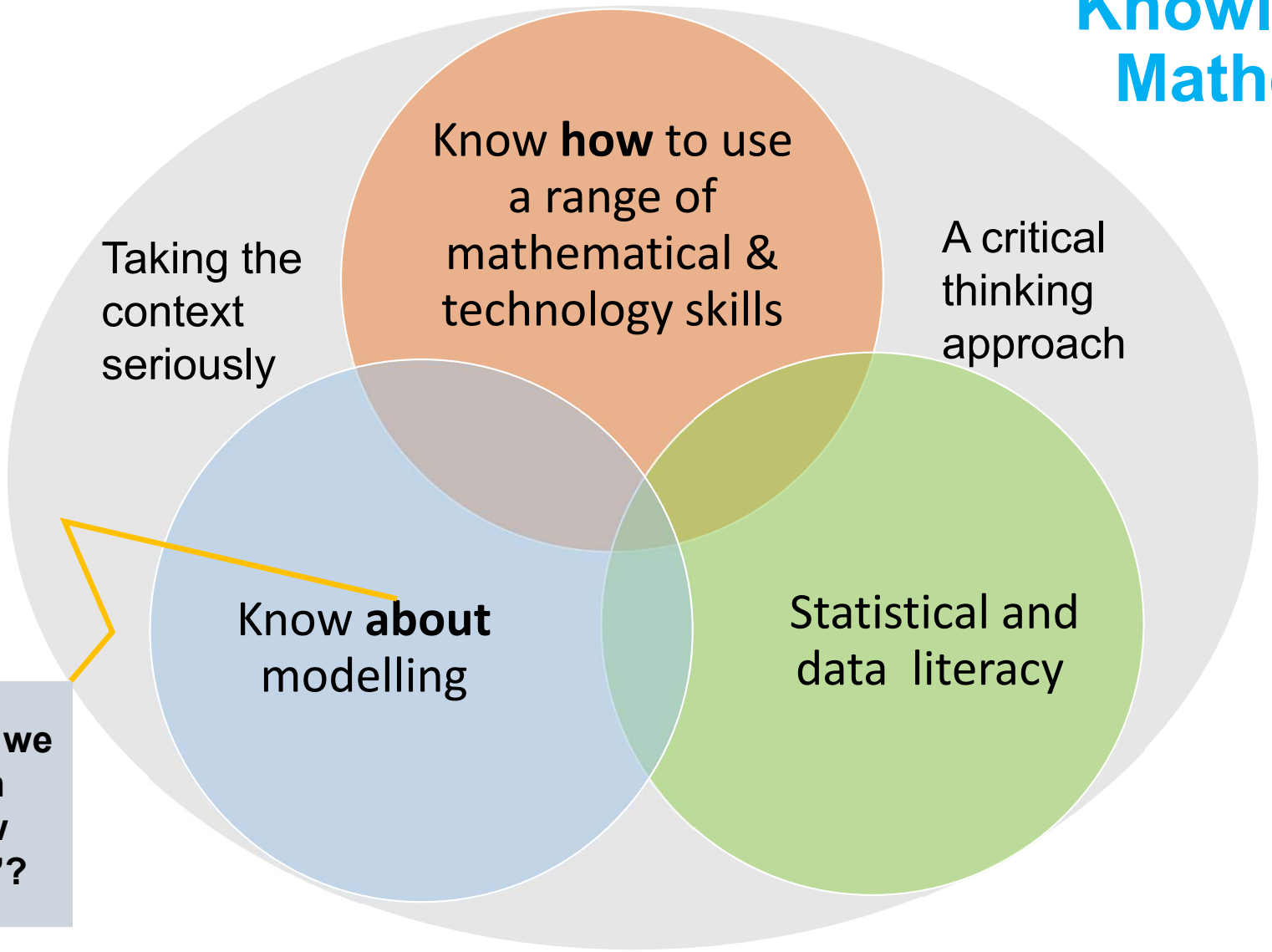
Climate change

Equality and inequality

Your money in your life

DESIGNING MEASURES

Knowledge for Mathematical Literacy



How do we teach 'know about"?

What does everyone need to know about modelling?

- That well-constructed mathematical models are frequently the best way we have to predict the future.
- That “all models are wrong but some are useful”
- That some models are incredibly complex (e.g. predicting the weather) and are only possible because of computers.
- That models depend on choice of variables, data and assumptions.

- That we encounter mathematical models and/or their results every day
- That most of the measures of anything other than directly physical quantities are models.
- That most people will not make their own mathematical models, but we all need to use them.

Some models predict the future very well!

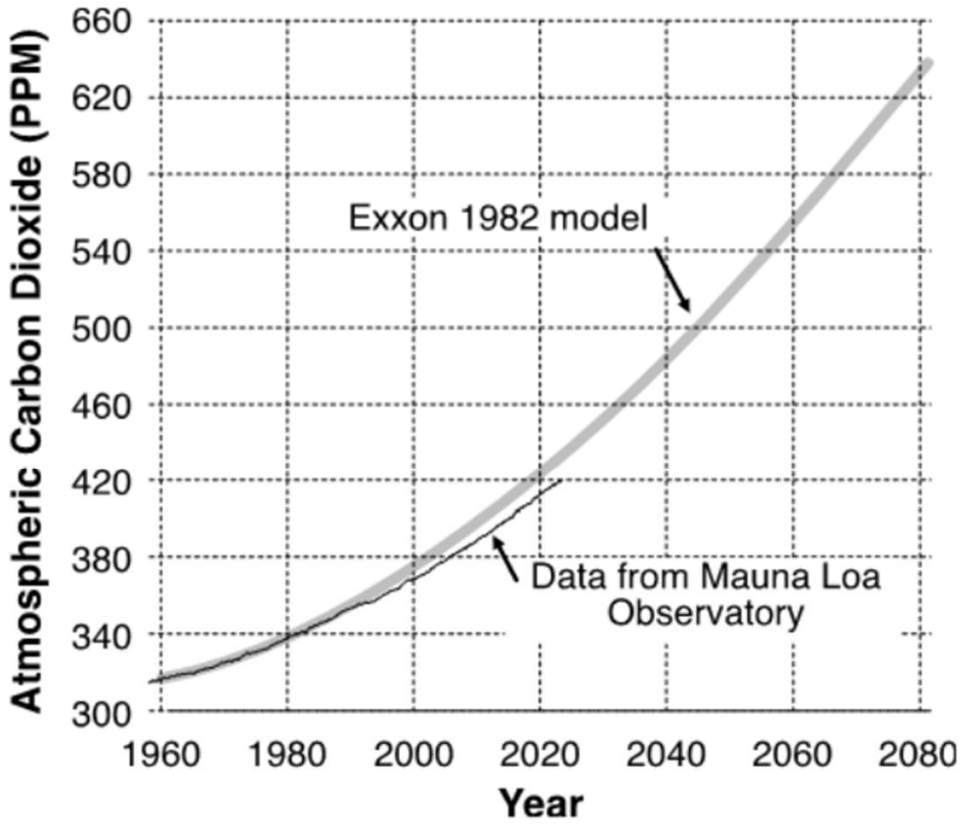


Figure 5.3.1a – Atmospheric concentration of CO_2 – predicted vs. measured

Data from ExxonMobil's predictions (Supran, 2023 [/5D]) and NOAA measurements [/5E]

Assessing Exxon Mobil'sGlobal Warning Projections
https://www.science.org/doi/10.1126/science.abk0063

LTML Supporting Examples website
ltml.mathlit.org

Models 1
Rules of Thumb

Bushwalking

Mathematical Modelling: Introduction | reSolve

Naismith's Rule estimates the time to allow for a walk:

* allow one hour for every 5 kilometres on the map plus

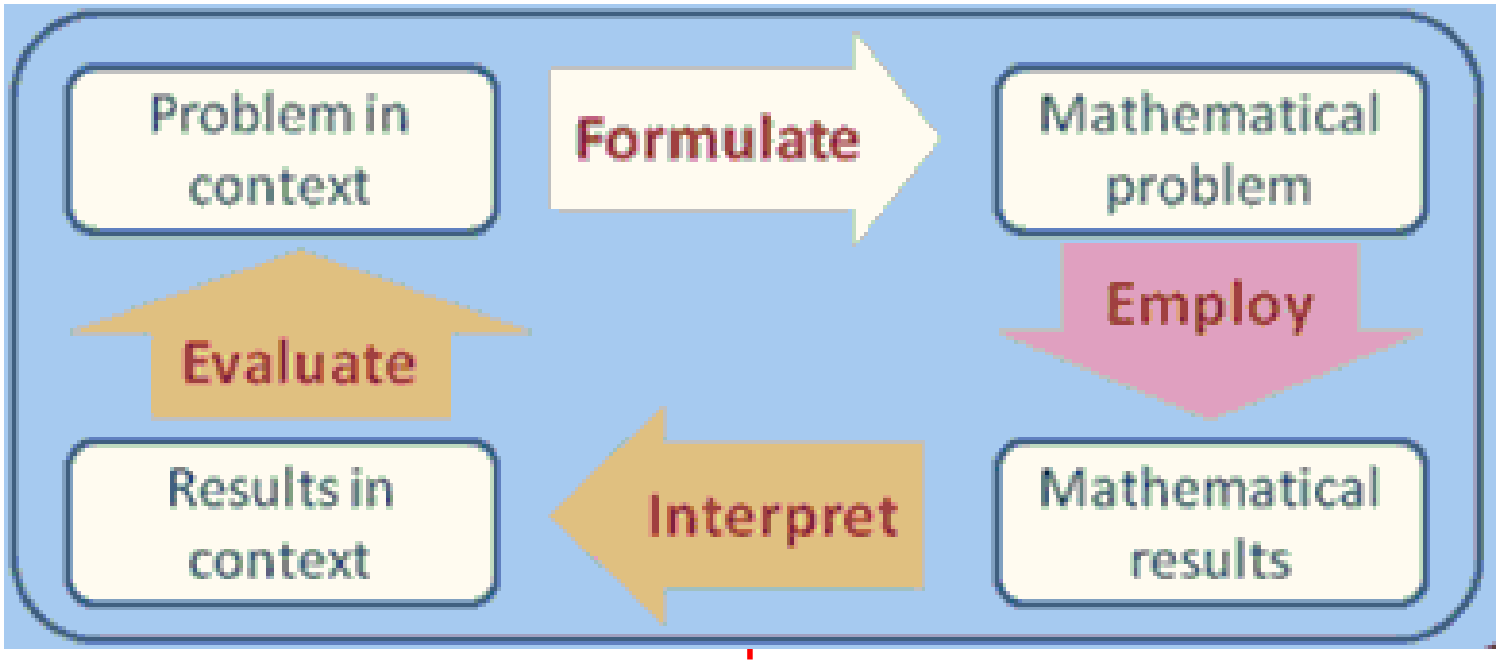
* 1 hour for every 600 metres of ascent.

Naismith's time in hours = (distance in km)/5 + (ascent in metres)/600

The modelling cycle

Thinking about real world ideas

Thinking about mathematical ideas



Models 1
Rules of Thumb

- What other “rules of thumb” do you or your students use, that are ‘mathematical models”?
- What variables are included and omitted in these rules?
- Any interesting mathematical relationships used?
- What real world understandings do you need to use them well?

Models 2
Ratings and Rankings

Simplest ratings system

People rank on 5 star system
Rating is the average

From Choice Australia website

A mathematical model of ‘goodness’ of washing machines

$G = 0.4d + 0.2r + 0.15g + 0.15w + 0.1s$

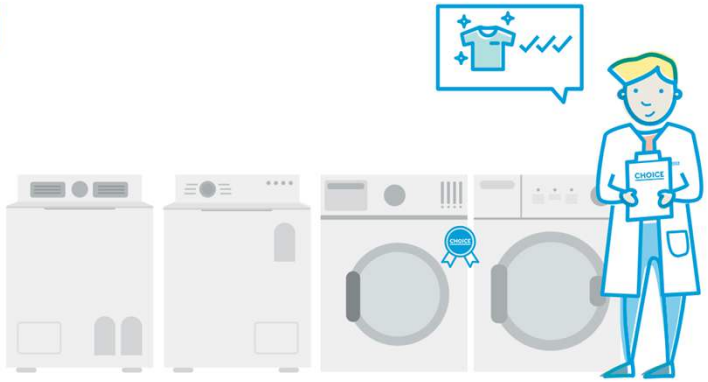
This is an example of a ‘utility function’ (linear function of many variables)

Washing machine reviews

Best washing machines to buy in Australia

Shop smarter and faster with our unbiased, expert washing machine reviews.

Compare now from $29.95



We give each washing machine an expert score, made up of:

- Dirt removal (40%)
- Rinse performance (20%)
- Gentleness score (15%)
- Water efficiency (15%)
- Spin efficiency (10%)

Models 2
Ratings and Rankings

- What other “rating systems” do you or your students use, that are ‘mathematical models”?
- What variables are included and omitted in these systems?
- What important / interesting mathematical features make these rating systems work well?
- What real world understandings do you need to use them well?

Activity

Design a utility function (variables with weights) to determine the GOAT for a sport or music style.

Appropriate weighting of variables
Scaling to make variable measures comparable
Sometimes eliminate outliers (e.g. Olympic diving)

Scoring the Decathalon

- Ten events – some fastest time, some furthest distance
- Want scores to represent ‘standard’ over time
  - So not just rankings

Scoring the Decathalon

Decathalon: The Art of Scoring Points
By John Barrow (https://nrich.maths.org/8346)

- Ten events – how to combine to make one score to decide who is the winner
  - Some running events where shortest time wins
  - Some throwing and jumping events where greatest distance wins
- Want a 'standard' over time.
  Various scoring methods devised since 1912, starting with 1000 points for world record times.
- Discusses options and evaluates against previous performances.

$$\text{Track event points} = \text{A} \times (\text{B} - \text{T})^{\text{C}}$$

$$\text{Field event points} = \text{A} \times (\text{D} - \text{B})^{\text{C}}$$

T time, D distance, A, B, C, selected constants

The recent rise of measurement
Models for constructive citizenship

If you can't measure it, then you can't manage it.
- Peter Drucker 1995 macroeconomics

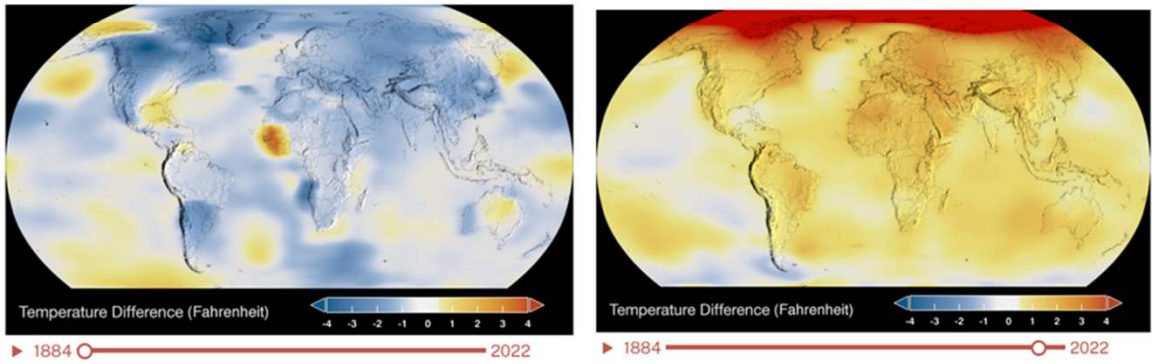
- When a measure becomes a target, it ceases to be a good measure.
  - Goodhart's Law 1975

- All models are wrong but some are useful!
  - George Box 1979

Models 3
Science, Health and Environment
Models for constructive citizenship

Time Series Animation
1884 to 2022
https://climate.nasa.gov/vital-signs/global-temperature/?intent=121

Areas of world cooler (blue) or warmer (red) than average year.



Species Diversity

- Why measure species diversity?
  - To make environmental decisions, monitor environmental plans
- As a measure of diversity, it has social use as well.

STATOLOGY

https://www.statology.org/simpsons-diversity-index-calculator/

Simpson's Diversity Index is a way to measure the diversity of species in a community.

To calculate this index for a given community, simply enter a list of observed frequencies for up to 10 species in the boxes below, then click the "Calculate" button:

Simpsons' Diversity Index

- Diversity relates to
  - Species richness
  - Species evenness

- Basic measure: probability that two organisms selected at random in the area are from the same species

Good measures have to use data that is easy enough to collect.

Duncan, Lenhart, Sturner (2014) Measuring biodiversity with probability. Mathematics Teacher 107(7), 547 – 552.

Royal Geographical Society – resources for schools

Models 4
Government, Economics, Social Policy
Models for constructive citizenship

Many examples:

GDP
Cost of Living
Inflation
Wage Growth
……..

MONEY

Working for free – some of us

VICTORIA DEVINE

A dark day on Australian calendars came around recently. It's not officially marked down and no alerts are issued. But for 48.7 per cent of the workforce who happen to be female, November 18 marked a significant moment in 2024.

It was the day in which the gender pay gap officially kicked in, and women began working essentially for free until the end of the year.

Though a vocal minority of society loves to pretend the gender pay gap is a myth that women, economists, statisticians and policy experts dreamed up one day, mountains of evidence shows otherwise, both here in Australia and around the world.

Overall, 2024 has been a profound year for discussing and coming to terms with the gender pay gap. In February, laws came into effect that triggered the Workplace Gender Equality Agency (WGEA) to publish specific gender pay gaps data for specific industries concerning Australian companies with more than 100 employees.

This meant that, for the first time, workers had a clear picture of how their employer and industry

Even with new efforts from the government, the gender pay gap remains stark in many industries.

Rather, it's about quantifying the difference in earnings between men and women in the workforce, and it is done by comparing the average pay for both genders.

Another positive result was the rate at which the average gap

Statistics that measures the average weekly ordinary time earnings of full-time employees aged over 18. In dollar terms, that's women earning 0.89 cents for every $1 men earn, equating to $231.50 less in a woman's pay-

gap based on these factors, it found the average pay gap across the 2023-24 year was 21.8 per cent, and the median was 18.3 per cent.

In its Equality Scorecard, published this month, the agency put it clearly: "For every $1 a man

factors at play are women being more likely to take time out of their careers to have children, and being more likely to work part-time.

But research published by economic institute e61 in May challenged this, showing that women were likely to be paid 15 per cent less than their male colleagues, even when their levels of education, age and family life are the same.

Still, there are some positives. In most female-dominated industries such as education and healthcare, the median pay gap is just 2.4 per cent, and the average is 5.5 per cent. And in gender-balanced industries such as retail, real estate, travel and hospitality, the median is 9.1 per cent.

But even where it is smaller, major room for improvement exists. In midwifery, for example, where the workforce is 99 per cent female, women are still paid 19 per cent less than their male colleagues.

Similarly, female-focused retail companies such as Lorna Jane and Pandora recorded gender pay gaps of 37 per cent and 52 per cent respectively – citing the predominance of men in higher-earning management and executive roles as the reason for such substantial gaps.

Feel like screaming after reading that? Me too. If there's a silver lining, it's that despite how dire the picture may seem, things are getting better.

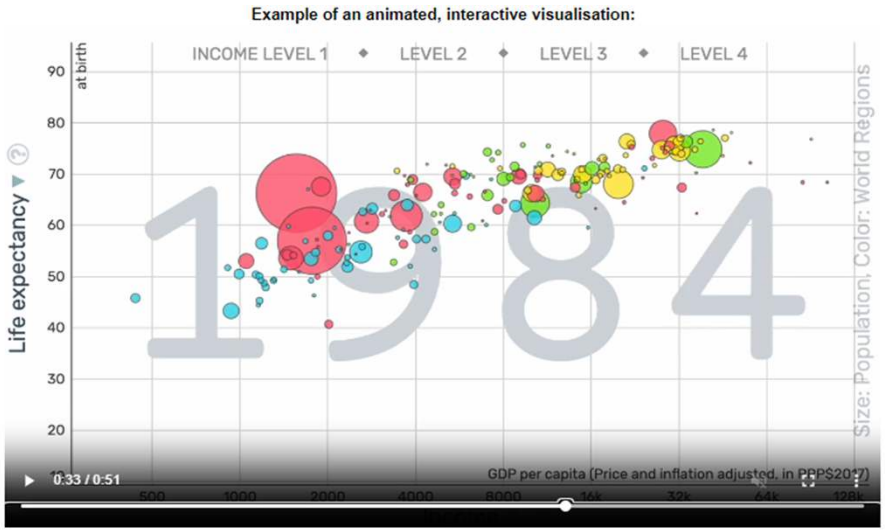
Next year, the data, reports and

SOCIAL EQUALITY AND INEQUALITY

What to measure?
How to define and calculate it?
What the data means?

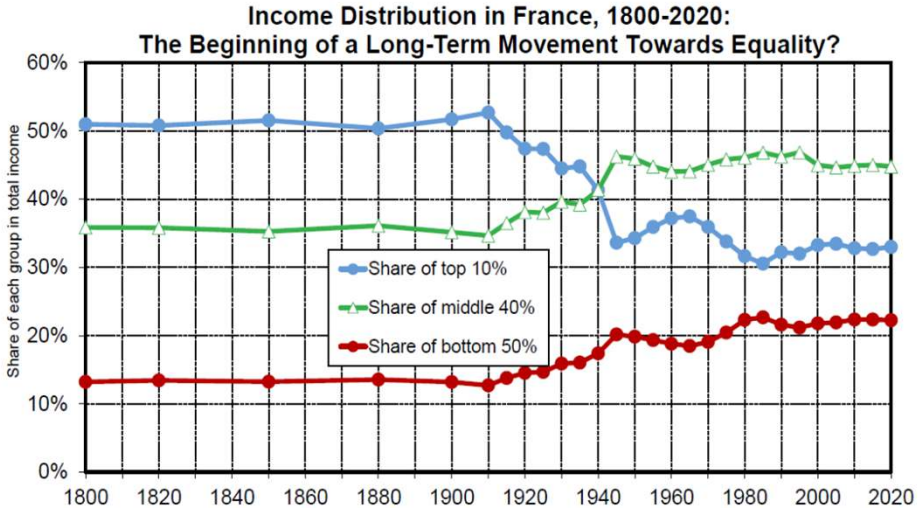
Gapminder Website

Gapminder.org (Rosling, Rosling & Ronnlund)



Thomas Piketty

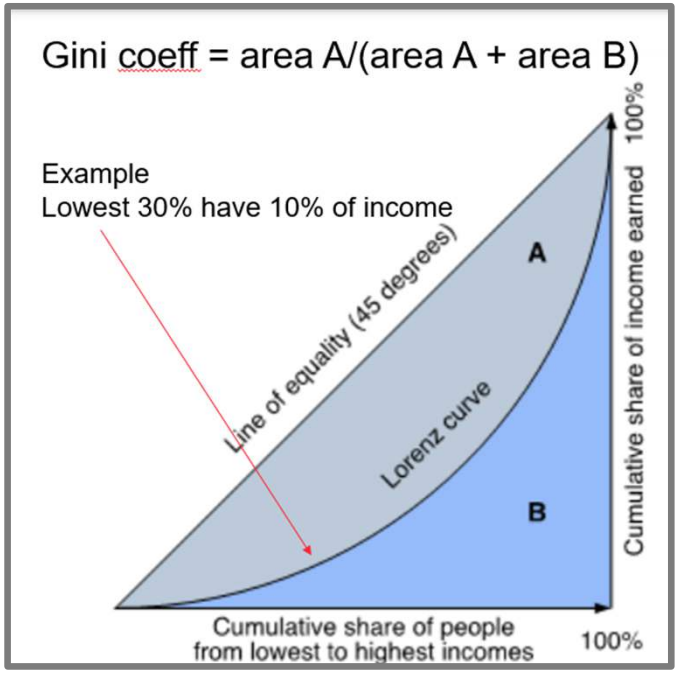
http://piketty.pse.ens.fr/fr/equality



Measures of SOCIAL EQUALITY AND INEQUALITY
The Gini Coefficient

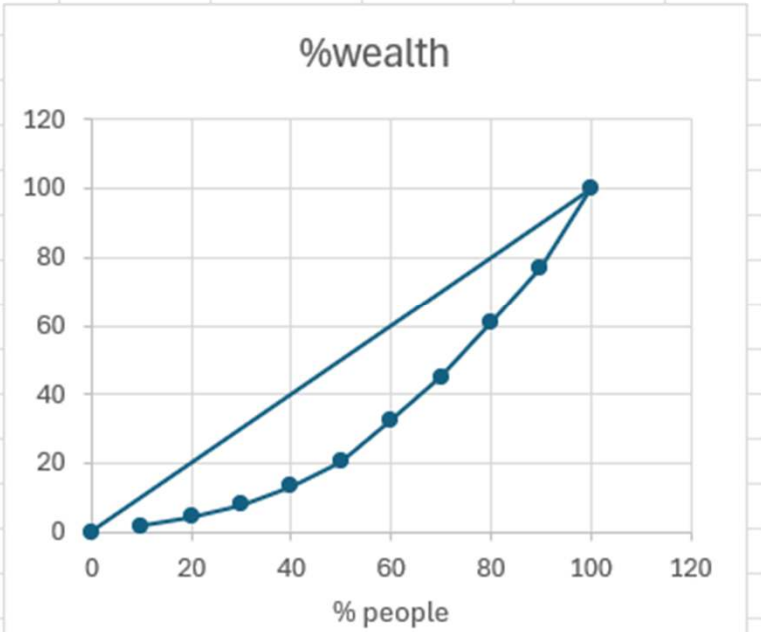
- Basic Idea – average unsigned difference between income of pairs of individuals, as a fraction of average income

- G = 0 everyone has same income
- G=1 someone has all the income
- G = ½ if half population have same high and half have same low income



Lorenz Curve for sample data

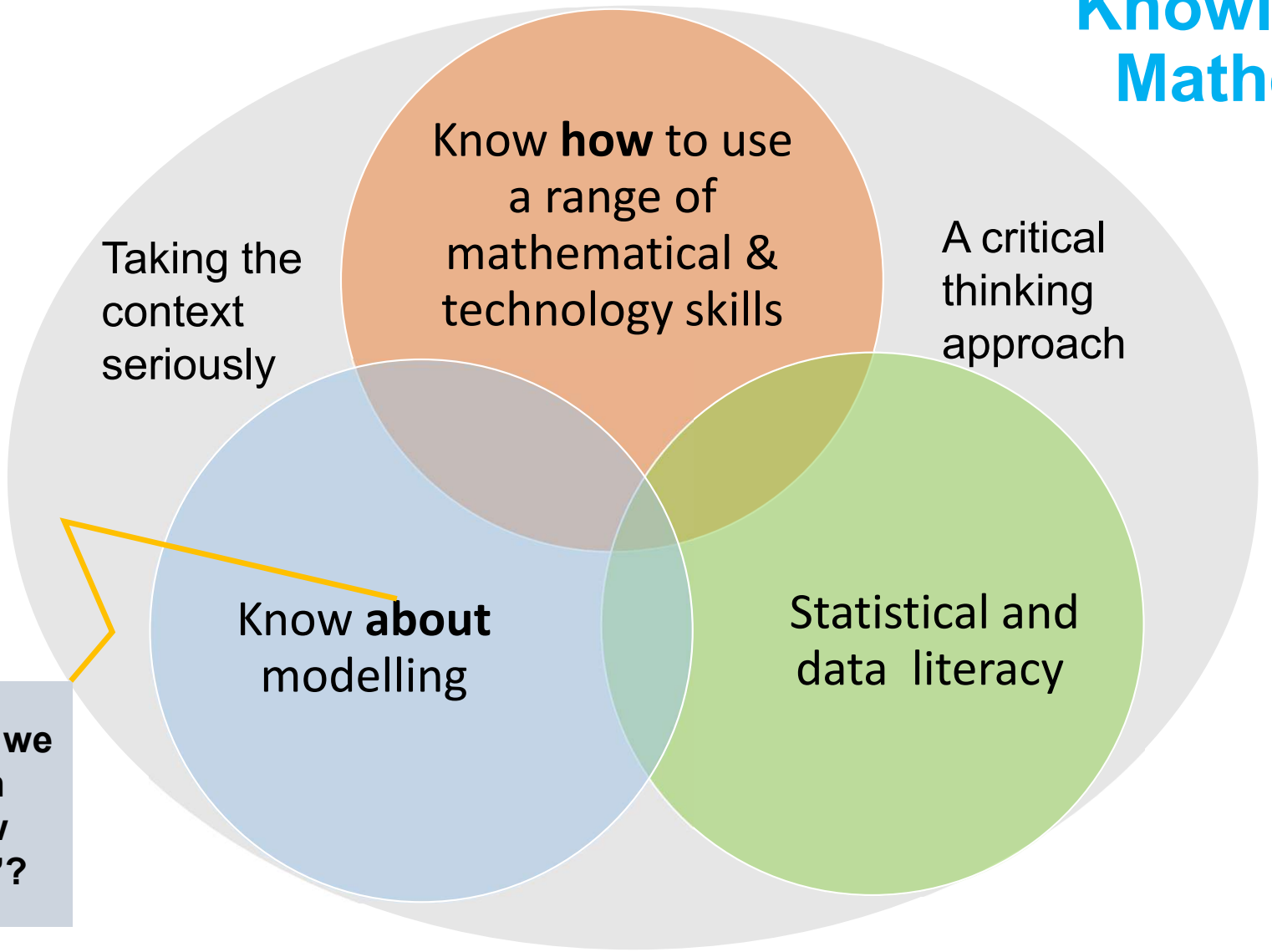
	A	B	C	D	F	G
1		Person	Wealth	Cumulativ	% people	%wealth
2		1	1	1	10	1.76678
3		2	1.5	2.5	20	4.41696
4		3	2	4.5	30	7.95053
5		4	3	7.5	40	13.2509
6		5	4	11.5	50	20.318
7		6	7	18.5	60	32.6855
8		7	7.1	25.6	70	45.2297
9		8	8.8	34.4	80	60.7774
10		9	9.2	43.6	90	77.0318
11		10	13	56.6	100	100
12	Total wealth		56.6		0	0
13						
14						
15						



Excel sheet to create Lorenz Curve

Microsoft Excel
)7-2003 Workshee

Knowledge for Mathematical Literacy



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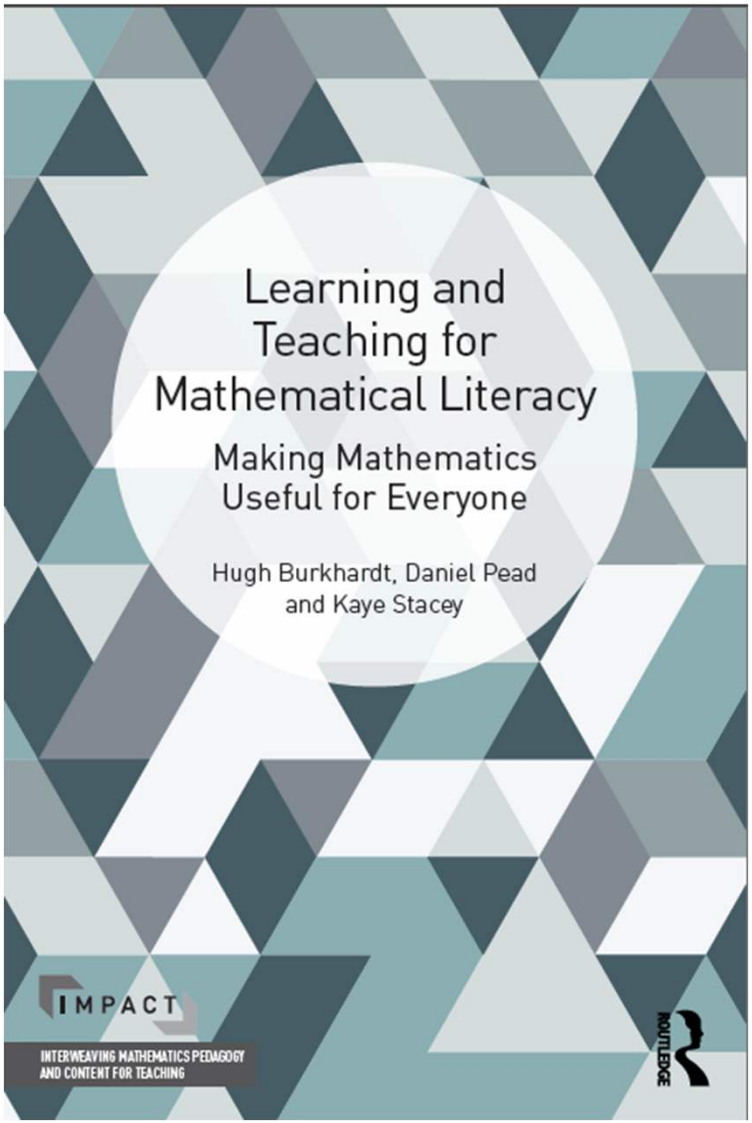

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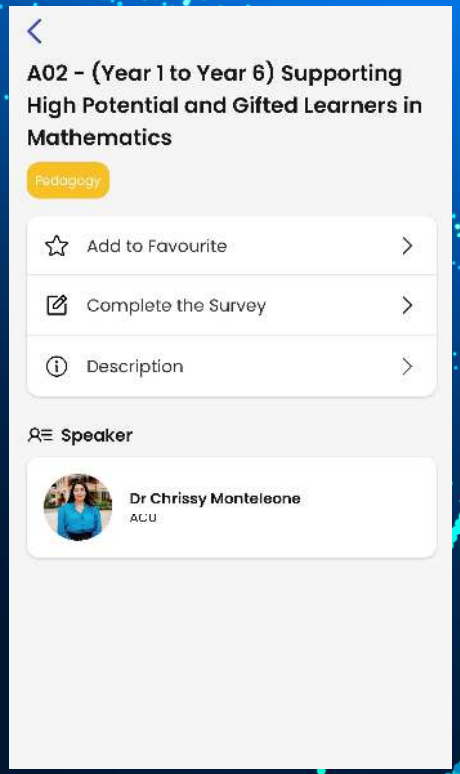
Be in it to WIN!

Thank you

Kaye Stacey
k.stacey.unimelb.edu.au



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Event App

App Download Instructions

Step 1: Download the App 'Arinex One' from the App Store or Google Play

App Store

Google Play

Step 2: Enter Event Code: **mav**

Step 3: Enter the email you registered with

Step 4: Enter the Passcode you receive via email and click 'Verify'. Please be sure to check your Junk Mail for the email, or see the Registration Desk if you require further assistance.

STATOLOGY
https://www.statology.org/simpsons-diversity-index-calculator/

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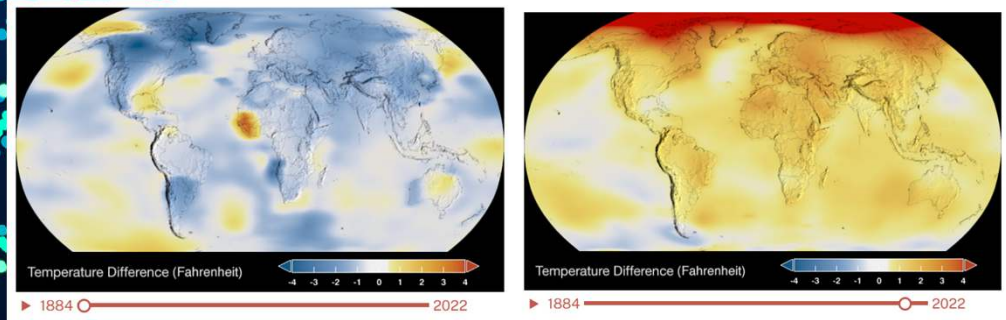
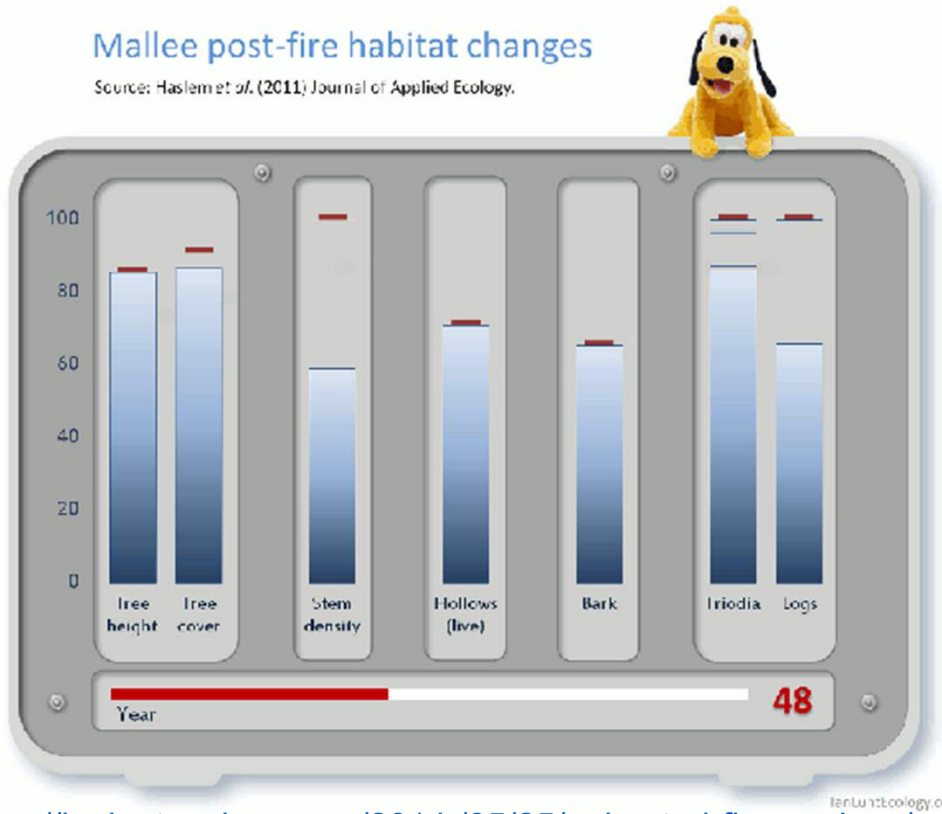
Time Series Animation
1884 to 2022
https://climate.nasa.gov/vital-signs/global-temperature/?intent=121

Areas of world cooler (blue) or warmer (red) than average year.



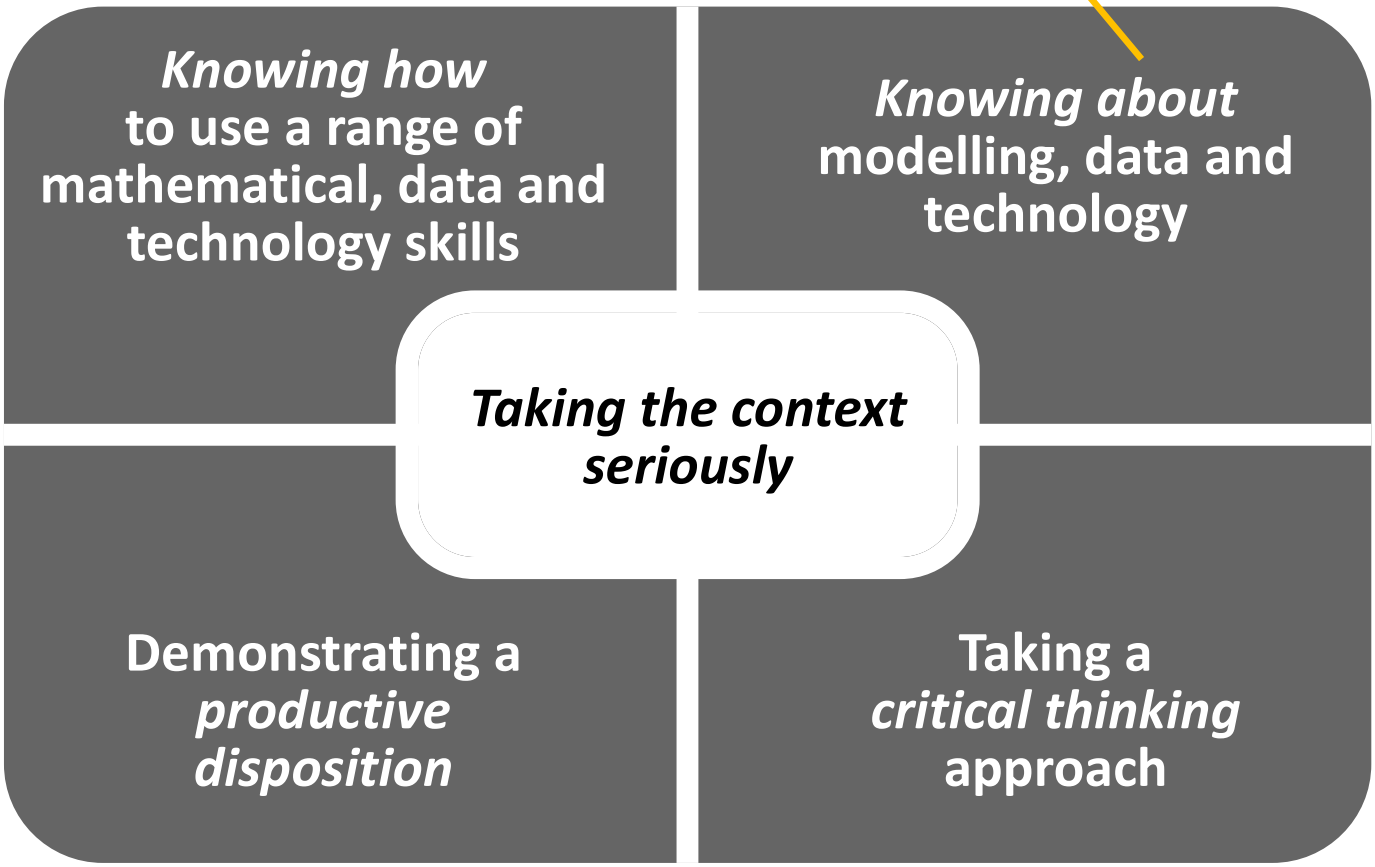
ENVIRONMENT

Delma australis



https://ianluntecology.com/2014 /05/25/animated-fire-ecology/

The Context-Focused Mathematics Framework



How do we teach 'know about"?

Literacy in context component

MULTIPLE EVERYDAY LIVING CHAPTERS:

MONEY, PLANNING, SPIN, USING COMPUTERS

- Using spreadsheets
- Assessing risk
- Saving and spending
- Inflation, interest rates and investments
- Understanding product ratings (utility functions)
- Aspects of 'spin' and advertising
- Planning events
- Making a good database
- Choosing a hard-to-break password
- Basics of AI and machine learning

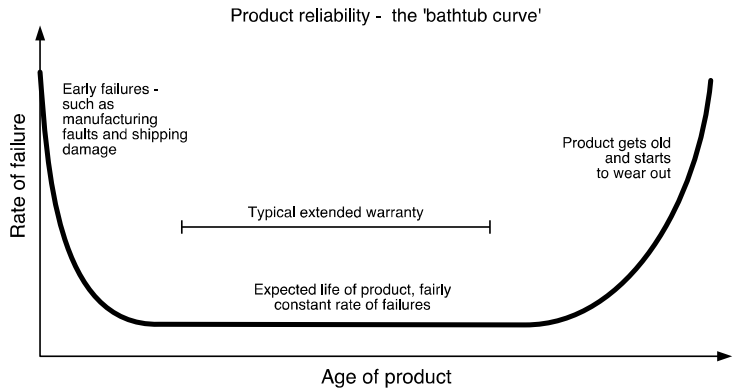
Activity

Design a utility function (variables with weights) to determine the GOAT for a sport or music style.
(GOAT = greatest of all time)

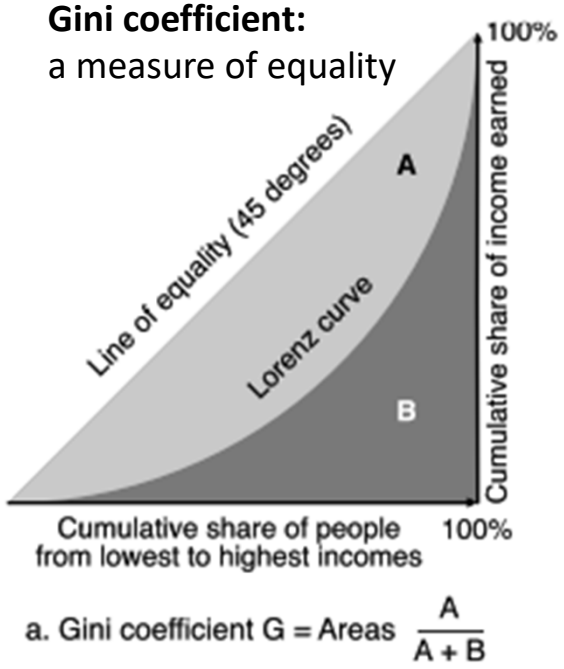
Mathematical modelling for measuring the world

Literally examples everyday

- Scoring Olympic events
- ‘Car of the Year’
- BMI
- GDP
- insurance, etc



Gini coefficient:
a measure of equality



a. Gini coefficient G = Areas $\frac{A}{A+B}$

Slide title goes here

- Body text goes here.
- To add more slides, click the down arrow next to ‘new slide’ and select ‘slide with title’

Slide with picture

Subheading

Body text goes here.

Slide with two columns of content

Four processes

- Mathematical modelling
- Computational Thinking
- Statistical Investigation
- Probability exp’ts and simulations

Recent curriculum changes

- Australian Curriculum: Mathematics (version 9, 2022)
  - first major revision since version 1 (2011)

- Victorian F -10 curriculum (and VCE subjects)
  - Victorian Curriculum - Mathematics 2.0 (vcaa.vic.edu.au) (2023)

- VCM2.0 aims include that students
  - develop useful mathematical and numeracy skills for everyday life and work, as active and critical citizens in a technological world
  - become confident, proficient, effective and adaptive users of mathematics

What does everyone need to know about modelling?

- That we encounter mathematical models and/or their results every day.
- That well-constructed mathematical models are frequently the best way we have to predict the future.
- That models depend on choice of variables, data and assumptions.
- That “all models are wrong but some are useful”
- That some models are incredibly complex (e.g. predicting the weather) and are only possible because of computers.
- That most of the measures of anything other than directly physical quantities are models:

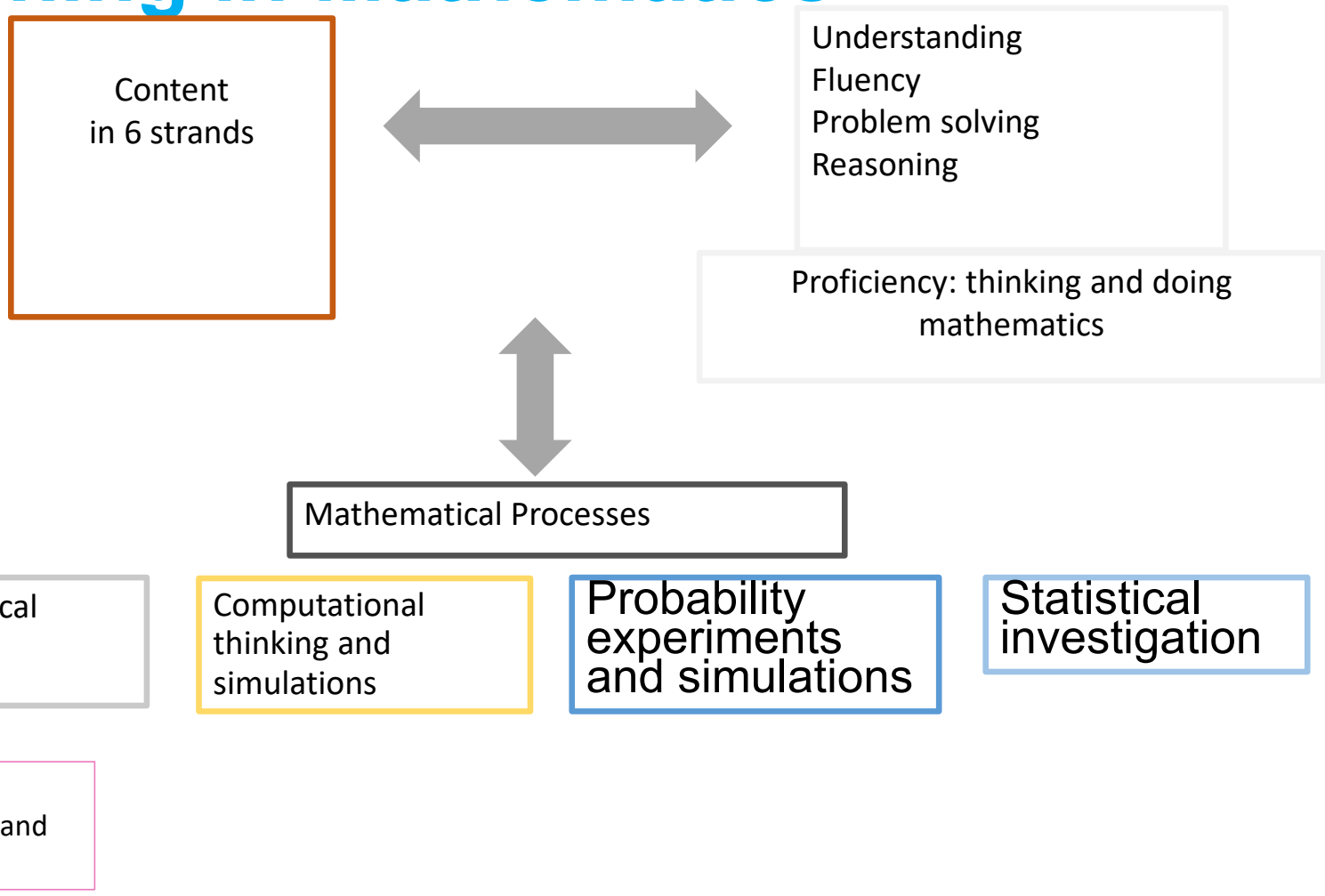
- BMI, GDP, CPI, ATAR, star ratings, social inequality, current size of Australia’s population, loan repayment apps,
  - That most people will not make their own mathematical models, but we all need to use them.
  -

Where is modelling in VCM2.0?

- **From Year F**. many Number content descriptions beginning “use mathematical modelling to solve practical problems involving . . . .
- modelling practical situations involving percentages using efficient calculation strategies …and interpreting the results in terms of the situation, for example, purchasing items during a sale
- modelling situations involving …budgeting, asking questions such as ‘Can I afford it?’ ….
  - **From Yr 7**, content descriptions also specify modelling steps and include Algebra and Measurement
    - e.g., formulate, justify choices.. etc
    - identifying variables in formulas, choosing functions, some science contexts (e.g., carbon dating)

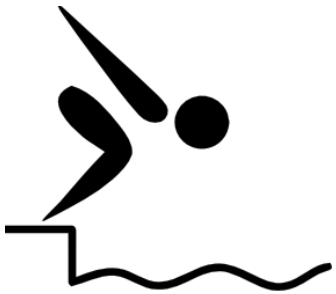
    From Yr 10, some more open modelling suggested and impact of measurement error e.g.,
    - growth of animal populations with varying reproductive behaviour (VC2M10A15),
    - collecting rainfall on a roof (VC2M10M01),
    - software model of 3-D crime scene.

Learning in Mathematics



Synge, 1951, p. 98

- The use of applied mathematics in its relation to a physical problem involves three steps.
  - First, a dive from the world of reality into the world of mathematics;
  - two, a swim in the world of mathematics;
  - three, a climb from the world of mathematics back into the world of reality, carrying the prediction in our teeth.



Australian Curriculum (v1, 2011)

Levels F to 10(A)

Three strands (13 substrands)

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability

Four embedded proficiencies

- **Understanding**
- **Fluency**
- **Problem Solving**
- **Reasoning**

Cross-Curriculum Priorities and General capabilities (incl numeracy, digi lit, crit thinking)

Australian Curriculum (v9, 2022)

Levels F to 10

Six Strands

- Number
- Algebra
- Measurement
- Space
- Statistics
- Probability

Four proficiencies as in AC:M(v1)

Four processes

- Mathematical modelling
- Computational Thinking
- Statistical Investigation
- Probability exp'ts and simulations

3 Priorities & 7 capabilities as v1.

Victorian Curriculum 2.0 (Vic, 2023)

Levels F to 10A (& early chhd)

Six strands

- Number
- Algebra
- Measurement
- Space
- Statistics
- Probability

Four proficiencies as in AC:M(v1)

Four processes as in AC:M(v9)

Priorities and capabilities organised somewhat differently

Australian Curriculum (v1, 2011)

Levels F to 10(A)

Three strands (13 substrands)

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability

Four embedded proficiencies

- **Understanding**
- **Fluency**
- **Problem Solving**
- **Reasoning**

Cross-Curriculum Priorities and General capabilities (incl numeracy, digi lit, crit thinking)

Australian Curriculum (v9, 2022)

Levels F to 10

Six Strands

- Number
- Algebra
- Measurement
- Space
- Statistics
- Probability

Four proficiencies as in AC:M(v1)

Four processes

- Mathematical modelling
- Computational Thinking
- Statistical Investigation
- Probability exp'ts and simulations

3 Priorities & 7 capabilities as v1.

Victorian Curriculum 2.0 (Vic, 2023)

Levels F to 10A (& early chhd)

Six strands

- Number
- Algebra
- Measurement
- Space
- Statistics
- Probability

Four proficiencies as in AC:M(v1)

Four processes as in AC:M(v9)

Priorities and capabilities organised somewhat differently

Four mathematical processes

Learning in Mathematics 2.0

- **Mathematical processes**
- Mathematical processes refer to the thinking, reasoning, communicating, problem-solving and investigation skills involved in working mathematically.
- Opportunities to learn process skills have been embedded across the strands, building in sophistication across the levels.
- Mathematical problem-solving and investigation draws on the processes of mathematical modelling, computational and algorithmic thinking, statistical investigation, probability experiments and simulations.
- **Mathematical modelling (one of four processes)**
- Mathematical models are used to gain insight into and make predictions about real-world phenomena, to inform judgements and make decisions in personal, civic and work life.
- In the modelling process students formulate a real-world problem mathematically by making assumptions; recognise, connect and apply mathematical structures; analyse and solve the mathematical model; and interpret, generalise and communicate their results in response to the real-world situation.
- Mathematical modelling is an essential dimension of the contemporary discipline of mathematics and is key to informed and participatory citizenship.

Modelling in the content descriptors

Year 8 VC2M8N06

use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts …; formulate problems, choosing efficient .. calculation strategies and using digital tools …; interpret and communicate solutions in terms of the context, reviewing the appropriateness of the model

Achievement standard. ….They use mathematical modelling to solve practical problems involving ratios, ….

Year 10 VC2M10A15

use mathematical modelling to solve applied problems involving inverse proportion, growth and decay, including in financial contexts to establish the compound interest formula as repeated applications of simple interest; formulate problems, choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings

The modelling cycle

Thinking about real world ideas

Thinking about mathematical ideas